Asymptotic delta-Parameterization of Surface-Impedance Solutions

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***Abstract*—The surface impedance methods are among the most efficient for solving time-harmonic eddy-current problems with a small penetration depth. When the solution is required for a range of frequencies, or material conductivities, the usual method leads to the solution of complex-valued problems for each frequency or conductivity. Here it is shown that a close method, issued from the asymptotic expansion, provides results parameterized by δ, with a comparable accuracy, but a reduced computational cost for a range of values for δ.**

***Index Terms*—surface impedance, asymptotics, parametric solutions.**

I. Surface impedances

The usual surface impedance method enables to solve approximately the problem of currents induced by a time-harmonic field in a conductor with a linear magnetic property, when the skin depth δ is small compared to the characteristic length of the device under study [1]. The solution is then performed only outside of the conductor (domain Ω). The existence of this conductor is taken into account by a specific condition on its boundary Σ

$$\mathbf{curl}\,\mathbf{H} = \mathbf{J}_{\text{source}} \text{ in } \Omega, \tag{1}$$

$$\mathbf{n} \times \mathbf{E} = Z_s \mathbf{n} \times (\mathbf{n} \times \mathbf{H}) \text{ on } \Sigma, \tag{2}$$

$$Z_s = (1 + \mathrm{j})/(\sigma\delta), \tag{3}$$

where σ and δ are respectively the electric conductivity and the skin depth. The solution by the FEM is straightforward; for instance in a 2d plane situation, the formulation in the A potential vector (A as J has only one component in this situation) gives :

$$-\Delta A = \mu_0 J_{\text{source}} \text{ in } \Omega; A = \alpha\delta\partial_{\mathbf{n}} A \text{ on } \Sigma; \alpha = \frac{\mathrm{j}-1}{2}. \tag{4}$$

If the frequency (or the conductivity) is modified, the solution has to be performed again.

II. Asymptotic expansion and parameterization

Instead of solving (4), a first approximate problem (5) and a sequence of problems (6) can be computed:

$$-\Delta A = \mu_0 J_{\text{source}} \text{ in } \Omega; A = 0 \text{ on } \Sigma. \tag{5}$$

$$-\Delta A_i = 0x \text{ in } \Omega; A_i = \partial_{\mathbf{n}} A_{i-1} \text{ on } \Sigma. \tag{6}$$

From these real solutions, an asymptotic expansion [2] in power of the small complex parameter $\alpha\delta$ can be considered

$$S = (\alpha\delta)^0 A_0 + (\alpha\delta)^1 A_1 + (\alpha\delta)^2 A_2 + \dots \tag{7}$$

The term A_0 being zero on Σ, from (6) and (7) we obtain

$$S = (\alpha\delta)^0 \partial_{\mathbf{n}} A_0 + (\alpha\delta)^1 \partial_{\mathbf{n}} A_1 + (\alpha\delta)^2 \partial_{\mathbf{n}} A_2 \text{ on } \Sigma \tag{8}$$

$$-\Delta S = -\Delta A_0 = \mu_0 J_{\text{source}} \text{ in } \Omega. \tag{9}$$

The whole asymptotic expansion (7) is thus the solution to (4). Once several terms A_0, A_1, ... has been computed (practically 2 or 3 are sufficient see Section III, the solution cost is reduced compared to (4) since the unknowns are real), the solution for any small δ can be reconstructed by a simple linear combination of these pre-computed solutions as in (7), without having to solve (4) for each value of δ. The gain in computational time can be consequent for sensitivity or parametric studies.

III. Numerical example

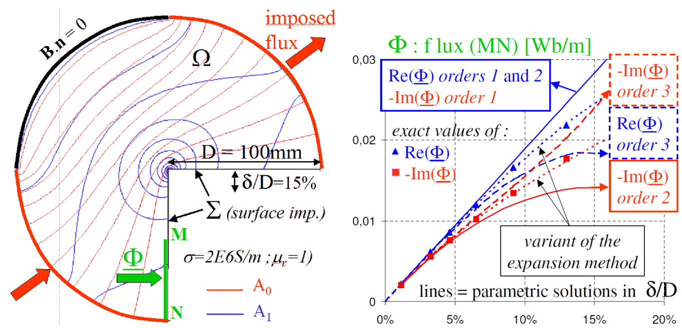

Fig. 1. Domain with A_0 and A_1 on the left. Flux vs. δ on the right.

A simple test case is introduced in Fig. 1. A flux is enforced that follows the surface Σ of a conducting angle. The firt two terms of (7) are shown in Fig. 1 left. In Fig. 1 right, the flux $\Phi = A(M) - A(N)$ through MN is shown as a function of δ. Compared to the exact results the approximat flux seems accurate for $\delta/D < 5\%$.

At the conference, we will detail a variant that enables with the same numerical cost (for order 3: 2 usual solutions with surface impedance) to widen the validity of the expansion ($\delta/D < 15\%$) thanks to a better correction effect due to the higher orders (see Fig. 1 right). We will also discuss error estimates, 3d formulation and the case of a linear magnetic conductor.

References

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